Counting

Pigeonhole Principle: Suppose you have n objects and assign k labels to these objects.
If $n > k$, then at least two objects must get the same label.

example) Naina, Charlotte, Aryan, Rahil must each be assigned to a room
there are 3 rooms: room A, B, C.

Naina, Charlotte, Aryan, Rahil $\longrightarrow$ room A.

example) Among the first 100 powers of 17, 2 of them must differ by a multiple of 57.

two numbers have the same remainder when divided by 57.

$17^1, \ldots 17^{100}$

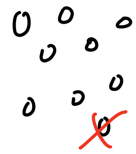
$\equiv a \pmod{57}$ $\quad \equiv b \pmod{57}$

There are only 57 possible remainders. (0-56)

Since we have 100 objects (powers of 17) that must be labeled with only 57 labels (remainders), by Pigeonhole principle, it must be the case that 2 of those objects have same label.

Permutations

Say we have 10 figurines we to line up in some order, How many ways can we order them?



$\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad}\ \underline{\quad} \cdot \underline{1}$

↑

$10 \cdot 9 \cdot 8 \cdot \ldots \cdot 1 = 10!$

What if we only have space for 4?

$$\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \qquad = 10 \cdot 9 \cdot 8 \cdot 7 = \frac{10!}{6!}$$

The number of permutations (or ordered choices) of k objects from n options is $\frac{n!}{(n-k)!} = P(n,k)$

"n permute k"

permutations w/ repetition:

$$C \quad O \quad \underline{L_1} \quad \underline{L_2} \quad \underline{E} \quad G \quad \underline{E}$$ ← 7 scrabble tiles

How many ways can we order them to make unique strings?

$$\left.\begin{array}{l} C \quad O \quad L_1 \quad L_2 \quad E \quad G \quad E \\ C \quad O \quad L_2 \quad L_1 \quad E \quad G \quad E \end{array}\right\}$$ these should be the same string

$$\frac{7!}{2!\,2!}$$

Combinations

10 figurines, choose 4 of them to pack

$$\frac{10!}{6!\,4!}$$

# of ways 4 figurines can be ordered.

$$C(n,k) = \frac{n!}{(n-k)!\,k!} = \binom{n}{k} = nCk$$

# of ways to choose set of k objects from n choices

combinations with repetition: I want to choose 6 plants for my garden from: thyme, oregano, mint.

In how many ways can I do this?

0 thyme

TT|O|MMM

||MMMMMM

|O|MMMMM

0 oregano

★★|★|★★★

||★★★★★★

number of ways to order 2 bars 6 stars gives us our answer

8 slots, 2 of them must be filled with bars.

$$\rightarrow \binom{8}{2} = \frac{8!}{6!\,2!} = \frac{8\cdot 7}{2} = 4\cdot 7 = 28$$

Choose k objects from n types. $= \binom{k+n-1}{n-1} = \binom{k+n-1}{k}$

$k = 6$, $n = 3$

☆ review binomial coefficients and binomial theorem ☆

↳ then do 17.6